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Context Aware Wireless Network

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The Team

• Rice
  • Brett Kaufman, Matthew Nokleby, Corina Ionita, David Ramirez,

• Oulu
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• Bandwidth \( W \)
• Time \( T \)
• Power
Applications

• Network with Infrastructure
• Cellular and WiFi
Applications

• Network with Infrastructure
  • Cellular and WiFi
• Ad hoc network
  • Rural
• First responders
• Machine to machine
  • Medical / Robotics
• Transportation
• Objective
  • Aggregate throughput
• Energy
• Physical layer view

• Scarce
  • Energy
  • Spectrum
• Physical layer view
• Scarce
  • Energy
  • Spectrum
• Cheap
  • Computation
• Memory
Wireless
Wireless

• Well understood
  • $DoF = \text{signal dimensions} = TW$
  • Reliable rate per $DoF = \log(1 + \frac{P_i d_i^{-\alpha}}{N_0})$
• Well understood
  • $\text{DoF} = TW$
  • Reliable rate per $\text{DoF} = \log(1 + \frac{P_i d_i^{-\alpha}}{N_0})$

• Required information
  • Minimal
  • LDPC, Turbo, Polar
• Less understood
• Network’s impact
• Well understood
• Not “optimum”
A Framework

- Share resources
• Partial abstraction

• Share
  • Time
  • Bandwidth
  • Spatial DoF
• Partial abstraction
• Share
  • Time
• Bandwidth
• Spatial DoF
• Questions
• Routing
• Scheduling
• Power control
• Modulation and coding
• Questions
• Routing
• Scheduling
• Power control
• Modulation and coding

Too hard!
Approach

- Granularity
- Session/flow
- Packet
- Symbol
- Signal
• Granularity
  • Session/flow
  • Packet
  • Symbol
  • Signal

• Examples
  • Spectrum sharing
  • Scheduling
  • Cooperation/network coding
  • Beamforming
- Granularity
  - Session/flow
  - Packet
  - Symbol
  - Signal

- Complexity
- Network information required
- Performance
• Granularity
  • Session/flow
  • Packet
  • Symbol
• Signal
Packet

- David Ramirez
- Multiple hops

\[
\log(1 + \frac{P_1 d_{12}^{-\alpha}}{N_0})
\]

\[
\log(1 + \frac{P_3 d_{34}^{-\alpha}}{N_0})
\]
Packet

- David Ramirez
- Multiple hops
- Full duplex nodes
Packet

- David Ramirez
- Multiple hops and multiple flows
- Full duplex nodes

\[ \log(1 + \frac{P_{i}d_{ij}^{-\alpha}}{\sum_{k} P_{k}d_{k,j}^{-\alpha} + N_0}) \]
Packet

- David Ramirez
- Multiple hops and multiple flows
- Full duplex nodes
  - Routing
  - Scheduling
  - Power control

\[ \log(1 + \frac{P_{i,d_{ij}}^{-\alpha}}{\sum_k P_{k,d_{kj}}^{-\alpha} + N_0}) \]
Packet

- David Ramirez
- Multiple hops and multiple flows
- Full duplex nodes
  - Routing
  - Scheduling
  - Power control

\[
\log(1 + \frac{P_{id}^{-\alpha}}{\sum_k P_k d_{kj}^{-\alpha} + N_0})
\]

Mixed integer/real non convex optimization
Packet

- Global network knowledge
- NP-hard

\[
\log(1 + \frac{P_i d_{ij}^{-\alpha}}{\sum_k P_k d_{k,j}^{-\alpha} + N_0})
\]

Mixed integer/real non convex optimization
Packet

- Global network knowledge
- NP-hard
- Decompose
  - Spatially
  - Temporally
- Clustering

Mixed integer/real non convex optimization
• Granularity
  • Session/flow
  • Packet
  • Symbol
  • Signal

• Complexity ↑
• Network information required ↑
• Performance ↑
Symbol

- Brett Kaufman
- Full duplex
- Interference channel
Symbol

- Brett Kaufman
  - Full duplex
  - High order modulation
  - Low density lattice coding
Symbol

- Brett Kaufman
- Full duplex
- High order modulation
- Low density lattice coding
- Cancelation
- Self interference
Symbol

• Brett Kaufman
• Full duplex
• High order modulation
• Low density lattice coding
  • Cancelation
  • Self interference
• Other flows
Symbol

- Brett Kaufman
- Full duplex
- System model
  - Analog RF cancelation
  - Baseband cancelation
    - Analog
    - Digital
Symbol

- Cooperation
- Network coding
Network Coding

- Linear network coding is “optimum”
Symbol

- Network information
- Node discovery
- Global channel state information
- Overhead and complexity
Network Coding with Relay

- Corina Ionita
Network Coding with Relay

- Corina Ionita

\[ w_1 \oplus w_2 \]
Network Coding with Relay

- Corina Ionita
Network Coding with Relay

- Corina Ionita
- Estimate
- Conditional expectation
- MAP detection
- Uncoded/coded
- Implementation on WARP
Network Coding

- Matt Nokleby
\[ f_1 = \bigoplus_{i=1}^{N} a_{i,1} w_i \]

\[ f_2 = \bigoplus_{i=1}^{N} a_{i,2} w_i \]
• Achievable “computation” rate [Nazer, Gastpar-2011]

• Lattice codes

• Only AWGN

• Fading?

\[ f_1 = \bigoplus_{i=1}^{N} a_{i,1} w_i \]

\[ f_2 = \bigoplus_{i=1}^{N} a_{i,2} w_i \]
• Network coding
• Fading
• Chained

\[
y = \sum_{i=1}^{N} h_i x_i + n
\]

\[
f = \bigoplus_{i=1}^{N} a_i w_i
\]

Computation rate \( \propto \log(\cdots < h, a >) \)
• Network coding
• Fading
• Chained

\[ y = \sum_{i=1}^{N} h_i x_i + n \]

\[ y_j = \sum_{i=1}^{N} h_{i,j} x_i + n_j \]
each other's message increases and the solution converges on further and further from each other. The burden of decoding computation rates in Figure 2 is due to the associated beamforming determined by $p_a$ combination rate, on the other hand, remains relatively high for each linear only be computed at low rate. The cooperative computation few linear combinations line up with the channels, so most can compute the computation rates for the first linear combinations.

In Figure 1, we place three users on a segment of the unit circle while the receiver is placed at the origin. We choose a block Markov encoding technique that achieves an improved performance for lattice-based physical-layer network coding. We have shown that user cooperation can improve performance and robust to mismatch between channels and the desired linear combination.

The non-cooperative rate drops off rather quickly. Only a few linear combinations line up with the channels, so most can be computed at low rate. We run $zyy$ simulations each for each user to the receiver are equal, this linear combination resulting in a block Markov encoding technique that achieves an improved performance for lattice-based physical-layer network coding. We have shown that user cooperation can improve performance and robust to mismatch between channels and the desired linear combination.

We sort the linear combinations in order of decreasing non-cooperative arclengths varying from $y$ to $z$. We run $zyy$ simulations each for each user to the receiver are equal, this linear combination resulting in a block Markov encoding technique that achieves an improved performance for lattice-based physical-layer network coding. We have shown that user cooperation can improve performance and robust to mismatch between channels and the desired linear combination.
In this section we examine a few example scenarios in which to demonstrate the benefits of our approach. The first example, depicted in Figure 5, comprises two transmitters randomly and uniformly on a segment of the circle having specified arclength. From the geometric configuration of the network, we compute channel gains between transmitters and receivers. As depicted in Figure 9, we again have close together. Even as we spread transmitters further apart, on average enough transmitters can cooperate that our approach garners a noticeable improvement. Again the trends are easy to appreciate. Cooperation offers the greatest improvement when transmitters are equally spaced.

For each realization we calculate the cooperative computation rate. Since the gains from transmitters to receiver are not equal, we place a single receiver at the origin and place or neither of them can; therefore we choose either $a = 1$ or $2$. The steering vectors and the clusters are optimized numerically. In Figure 6 we plot the achievable rate of our cooperative scheme against the upper bound of Theorem 2, $g_{ij}$, which we vary such that the gain $a = 4$. Since the channels between transmitters and receiver are not symmetric, with the forward coefficients constant, the steering vectors $\mathbf{v}_{0}$, $\mathbf{v}_{1}$, and $\mathbf{v}_{2}$. The optimal choice is the Euclidean distance between users $d_{i, j}$. We find the optimal tradeoff between diversity and multiplexing for compute-and-forward, and as we saw in Figure 2 the two approaches combined quickly converge on the upper bound. We note a "dimple" in the cooperative rate as $g_{ij}$ becomes large. For sufficiently large $a$, the Nazer-Gastpar component of the cooperative rate is zero, meaning that only the jointly-encoded resolution information carries information to the receiver. At this value of SNR at $P_{21}$, and plot the average computation rates in Figure 7. We place a single receiver at the origin and place

\[ g_{ij} = \frac{1}{d_{i, j}} \]

\[ v_{0}, v_{1}, v_{2} \]

\[ g_{ij} = \frac{1}{d_{i, j}} \]

\[ a = 4 \]

\[ v_{0}, v_{1}, v_{2} \]

\[ g_{ij} = \frac{1}{d_{i, j}} \]

\[ a = 4 \]

\[ v_{0}, v_{1}, v_{2} \]

\[ g_{ij} = \frac{1}{d_{i, j}} \]

\[ a = 4 \]

\[ v_{0}, v_{1}, v_{2} \]

\[ g_{ij} = \frac{1}{d_{i, j}} \]

\[ a = 4 \]
To Achieve

- Rates within a network
- Coding and modulation
- Medium to high SNR
To Achieve

- Rates within a network
- Coding and modulation
- Medium to high SNR

- Context
  - Interference
  - Cooperation
- Side information
To Achieve

- BPSK?
Background

- BPSK?
- Higher order modulation and coding

\[ \sin(\omega_c t) \quad \cos(\omega_c t) \]
Background

• Integers on a plane
• Lattice is a subgroup of $\mathbb{R}^2$

$\mathbb{Z}^n$ is a simple lattice.
Lattices

- An example of $BZ^2$ in $\mathbb{R}^2$
- Imagine $\mathbb{Z}^n$ and $\mathbb{R}^n$
- Closed under addition
- Symmetric

$\lambda_1, \lambda_2 \in \Lambda \implies \lambda_1 + \lambda_2 \in \Lambda$

$\lambda \in \Lambda \implies -\lambda \in \Lambda$

$BZ^n$
Decoding

- Decision boundary
- Hexagonal

The Voronoi region of a lattice point is the set of all points that quantize to that lattice point. The fundamental Voronoi region $V$ consists of points that quantize to the origin, $V = \{x: Q_\Lambda(x) = 0\}$.
Nested Lattice

- Lattice and sublattice

Nested Lattices

\( \Lambda \) and \( \Lambda_{FINE} \) are nested if \( \Lambda \subset \Lambda_{FINE} \).

Nested Lattice Code:

All lattice points from \( \Lambda_{FINE} \) that fall in the fundamental Voronoi region \( V \) of \( \Lambda \).

\[ \text{Rate} = \frac{1}{n} \log \left( \frac{\text{Vol}(V)}{\text{Vol}(V_{FINE})} \right) \]
Nested Lattice

Two lattices $\Lambda$ and $\Lambda^\text{FINE}$ are nested if $\Lambda \subset \Lambda^\text{FINE}$.

Nested Lattice Code:

All lattice points from $\Lambda^\text{FINE}$ that fall in the fundamental Voronoi region $V$ of $\Lambda$.

$V$ acts like a power constraint

$$\text{Rate} = \frac{1}{n} \log \left( \frac{\text{Vol}(V)}{\text{Vol}(V^\text{FINE})} \right)$$
• Power constrained
Lattice Coding

- Large dimensional coded modulation with power constraint
Lattice Coding

• Large dimensional coded modulation with power constraint
Lattice Coding

- Large dimensional coded modulation with power constraint
- Coding lattice is fine
- Shaping lattice is coarse

\[ \Lambda_s \subset \Lambda_c \]
Lattice Coding

- Large dimensional coded modulation with power constraint
- Coding lattice is fine
- Shaping lattice is coarse

\[ \Lambda_s \subset \Lambda_c \]
Lattice Coding

- Large dimensional coded modulation with power constraint
Lattice Coding

- Large dimensional coded modulation with power constraint
Lattice Coding

- Large dimensional coded modulation with power constraint

Diagram:
- Coding lattice
- Shaping lattice
- Fundamental Voronoi region
Lattice Coding

- Large dimensional coded modulation with power constraint
Lattice Coding

- Large dimensional coded modulation with power constraint

\[
\Lambda_s \subset \Lambda_c \\
\text{Vol}(\mathcal{V}_s) > \text{Vol}(\mathcal{V}_c)
\]
Lattice Coding

- Large dimensional coded modulation with power constraint

\[ \Lambda_s \subset \Lambda_c \]
\[ \text{Vol}(\mathcal{V}_s) > \text{Vol}(\mathcal{V}_c) \]
Lattice Coding

- Large dimensional coded modulation with power constraint Lattice codes
- Isomorphism

\[ w \leftrightarrow \lambda \mod \Lambda_s \]
Lattice Coding

- Large dimensional coded modulation with power constraint Lattice codes
- Isomorphism

\[ \phi(w) = \lambda \]

\[ w \leftrightarrow \lambda \mod \Lambda_s \]
System

\[
\begin{align*}
&\text{Encoder} \quad \text{Male} \\
&\quad x \quad + \\
&\quad y \quad \text{Mod} \\
&\quad \tilde{y} \quad \text{Decoder} \\
\end{align*}
\]
System
Rate and Error

- The probability of error $\mathbb{P}\{w \neq \hat{w}\} \rightarrow 0$ as $n \rightarrow \infty$
- The rate

$$\text{Rate} = \frac{1}{n} \log \left( \frac{\text{Vol}(\mathcal{V}_s)}{\text{Vol}(\mathcal{V}_c)} \right)$$

Diagram:

- Encoder
- Mod
- Decoder
- Shaping
- Coding
Rate and Error

- The probability of error \( \mathbb{P}\{w \neq \hat{w}\} \to 0 \) as \( n \to \infty \)
- The rate

\[
\text{Rate} = \frac{1}{n} \log \left( \frac{\text{Vol}(\mathcal{V}_s)}{\text{Vol}(\mathcal{V}_c)} \right)
\]

Diagram:
- Encoder
- Shaping
- Mod
- Coding
- Decoder
- \( w \)
- \( x \)
- \( y \)
- \( \hat{y} \)
- \( \hat{w} \)
Encoding

- Mapping from \( p \)-ary to integers
- Isomorphism \( x = G\lambda \mod \Lambda_s \)
Encoding

- Mapping from p-ary to integers $\phi(w) = \lambda$
- Isomorphism $x = G\lambda \mod \Lambda_s$
Linear Coding

• The generator matrix \( G \in \mathbb{R}^{n \times n} \)

• The parity check matrix \( H = G^{-1} \in \mathbb{R}^{n \times n} \)

• Sparse \( H \)
Low Density Lattice Codes (LDLC)

- Sparse non-binary parity check matrix
- Example [Sommer, Feder, Shalvi-2008]
- Regular latin square with degree 3

\[
H = \begin{pmatrix}
0 & -0.8 & 0 & -0.5 & 1 & 0 \\
0.8 & 0 & 0 & 1 & 0 & -0.5 \\
0 & 0.5 & 1 & 0 & 0.8 & 0 \\
0 & 0 & -0.5 & -0.8 & 0 & 1 \\
1 & 0 & 0 & 0 & 0.5 & 0.8 \\
0.5 & -1 & -0.8 & 0 & 0 & 0
\end{pmatrix}
\]
LDLC

- Encoding complexity linear in $n$
- Iterative decoding with complexity linear in $n$
- Bipartite graphical decoding [Sommer, Feder, Shalvi-2008]

$$Hx = HG\lambda$$
$$\sum_k h_k x_{i_k} = \text{integer}$$
LDLC

- Degree 5 for n=100 and 7 for others
- Iterations <200
- Symbol error rate [Sommer, Feder, Shalvi-2008]

[Symbol error rate (SER) for various block lengths]
LDLC

- Effective in medium to high SNR
- Relatively simple encoding and decoding
- Linear structure
- Superposition
- Decomposable
- Amenable to side information
LDLC in Networks

• Network coding

• Superposition [Nazer, Gastpar-2011]

\[ \lambda_i \mod \Lambda_s \iff w_i \]

\[ \lambda_1 + \lambda_2 \mod \Lambda_s \iff w_1 \oplus w_2 \]
\[
\sum_{i=1}^{N} a_i \lambda_i \mod \Lambda_s
\]
\[ \sum_{i=1}^{N} a_i \lambda_i \mod \Lambda_s \]
LDLC

- Decomposable \cite{Nokleby-Aazhang-2012}

\[
C = \Lambda_c \cap \mathcal{V}_s = \Lambda_r \cap \mathcal{V}_s + \Lambda_l \cap \mathcal{V}_s
\]
\[ \sum_{i=1}^{N} a_i \lambda_i \mod \Lambda_s \]
• Transmit in two stages
• User’s lattice code
• Resolution lattice code
• Linear combination
- Transmit in two stages
- User’s lattice code
- Resolution lattice code
- Linear combination

- Decode in stages
  - Resolution
  - Subtract
  - Leftover
LDLC in Networks

- Side information
- Interference
  - Full duplex
- Successive interference cancelation
- Layered coding
Final Thoughts

- Multi-flow networks
- Granularities
  - Session, packet, symbol, signal
- Applications
- Implementation on WARP